The Simplest Phased Array Feed System...That Works

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Many amateurs having a phased-array antenna use the feed system shown in Fig 1, with the difference between the electrical lengths of the feed lines equaling the desired phase angle. The result is often disappointing. The reasons for poor results are twofold.

1) The phase shift through each feed line is not equal to its electrical length, and

2) The feed line changes the magnitude of the current from input to output.

This surprising combination of events occurs in nearly all amateur arrays because of the significant, and sometimes dramatic, change in element feed-point impedances by mutual coupling. The element feed-point impedances—the load impedances seen by the feed lines—affect the delay and transformation ratio of the cables. It isn't a small effect, either. Phasing errors of several tens of degrees and element-current ratios of 2:1 are not uncommon. Among the very few antennas which do work are arrays of only two elements fed completely in phase (0°) or out of phase (180°). This topic is covered in detail in *The ARRL Antenna Book*.[1]

It is possible, however, to use the system shown in Fig 1 and have the element currents come out the way we want. The trick is to use feed-line lengths which give the desired delay and transformation ratio *when looking into the actual element impedances*. More specifically, we choose the feed-line lengths to give the desired ratio of currents, with the correct relative phasing. This paper explains how to find the correct feed-line lengths, and includes a BASIC program to do the calculations. (See the Program 1 listing near the end of this paper.) Table 1 gives results for a 90°-fed, 90°-spaced, 2-element array.

Calculation of the feed-line lengths, with or without the program, requires knowledge of the element self- and mutual impedances. Most of us don't know these impedances for our arrays, so one of several approaches can be taken.

1) Measure the self- and mutual impedances using the techniques described in *The Antenna Book* (see note 1). If carefully done, this approach will lead to the best array performance.

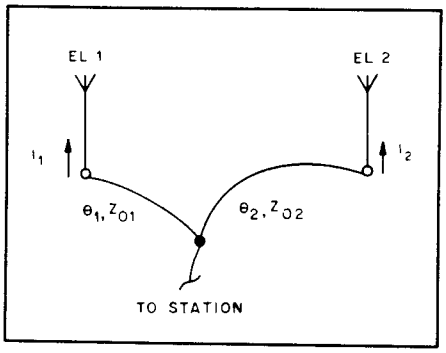

Fig 1—A typical feed system for two-element phased arrays. See text for a discussion of the feed-line lengths. Grounds and cable shields have been omitted for clarity.

2) Estimate the self- and mutual impedances. Methods and graphs are given in the *The Antenna Book*. This approach can lead to very good array performance if the array elements are straight and parallel, and with no loading elements or unusual features.

3) If the elements fit the above description, and in addition are self-resonant and close to ¼ λ high, you can use Table 1 instead of the program, if desired.

4) If you can't measure or estimate the self- and mutual impedances with reasonable accuracy, and your elements don't fit the description given in approach 3, you're likely to get poor results with this feed system. A better approach would be to use the L-network feed system described in *The Antenna Book*. It's quite simple and has the advantage of being adjustable. Adjustment methods also are given in *The Antenna Book*.

Using The Program

Program 1 was purposefully written in a very simple form of BASIC. It should run on nearly any computer without modification. If you encounter difficulty, the most likely cause is that the program was not copied exactly as printed.

The first prompts are for the self-R and X of the elements. These are the impedances which would be measured at the base of each element with the other element open-circuited at the base. The self-R includes any loss resistance. The remainder of the prompts are self-explanatory. Refer to Table 2, the sample run, for an example

Table 1

Phasing Line Lengths for a 90°-Fed, 90°-Spaced Two-Element Array

See Fig 1.

R_s, Ohms	No. Radials per Element	Phasing Lines: Z_0, Ohms		Elect Length (Deg)	
		1	2	Line 1	Line 2
65	4	50	50	No solution	
		75	75	No solution	
		75	50	30.36	104.96
				95.13	162.96
54	8	50	50	No solution	
		75	75	68.15	154.29
				132.60	184.95
45	16	50	50	No solution	
		75	75	58.69	153.48
				144.43	183.39
36	∞	50	50	80.56	154.53
				131.68	173.23
		75	75	51.61	155.40
				153.86	179.13

[1]Notes appear on page 29.

Table 2

Sample Run of Program 1 for a 90°-Spaced, 90°-Fed Array

Calculations are for resonant elements, approximately ¼-λ high, with 8 ground radials per element.

```
RUN
SELF R, X OF LEADING ELEMENT (OHMS)
? 54,0
SELF R, X OF LAGGING ELEMENT (OHMS)
? 54,0
MUTUAL R, X (OHMS)
? 20,-15
EL.2:EL.1 CURRENT MAGNITUDE, PHASE (DEGREES)
—PHASE MUST BE ZERO OR NEGATIVE
1,-90
FEEDLINE 1, 2 IMPEDANCES (OHMS)
? 50,50
NO SOLUTION FOR THE SPECIFIED PARAMETERS.
  WOULD YOU LIKE TO TRY DIFFERENT
  FEEDLINE Z0'S (Y,N)? Y
FEEDLINE 1, 2 IMPEDANCES (OHMS)
? 75,75
```

	Z0 = 75 OHMS TO LEAD. EL. ELECT. L. (DEG.)	Z0 = 75 OHMS TO LAG. EL. ELECT. L. (DEG.)
FIRST SOLN.	132.6038	184.9522
SECOND SOLN.	68.1518	154.2918

Ok

of program operation.

Sometimes you might get the result, NO SOLUTION FOR THE SPECIFIED PARAMETERS. This doesn't mean there's a solution which the program couldn't find; it means that there really is no solution for the specified conditions. If this happens, try different feed-line impedances. I've found a combination of common feed-line impedances which will work with nearly every array I've wanted to feed, but there are some which can't be fed using this method.

Whenever there is a solution, there's also a second one. Both are computed by the program. It may be necessary to use the longer set of feed-line lengths in order to make the feed lines physically reach the elements. You can also add ½ λ of cable to *both* feed lines and maintain correct operation. For example, the array in the sample program run of Table 2 can be fed with two 75-ohm lines of the following lengths (given in electrical degrees).

68.15° and 154.29°
132.60° and 184.95°
248.15° and 334.29°
312.60° and 364.95° (or 4.95°)

The first two sets are the lengths given by the program. A half wavelength is added to both lines to make sets 3 and 4. Note that a *full* wavelength can be subtracted from the second line length in the last set.

Occasionally it's necessary to make the feed-line impedances different from each other. If you want to be able to switch the pattern direction but have unequal feed-line impedances, add ½ λ of line from each element to the phasing feed line. If both ½-λ lines have the same impedance, directional switching will be possible while maintaining correct phasing.

Using the Table

Table 1 gives the feed-line lengths necessary to correctly feed a 90°-fed, 90°-spaced, 2-element array. The table is based on the following assumptions:

1) The elements are identical and parallel.

2) The ground systems of the elements have equal loss.

3) The elements are resonant when not coupled to other elements. A height of $237/f_{MHz}$ will be close to resonance for most vertical elements.

4) The elements are not loaded and do not have matching networks at their bases. Traps generally act like loading elements on the lower bands.

5) The elements are fairly "thin." HF antennas made from wire, tubing, or common tower sections fit this category.

6) Your ground isn't unusually dry or swampy. If it is, you may have more or less element self-resistance than shown for the number of radials. The resistance versus number of radials is based on measurements by Sevick.[2]

Since so many factors can affect ground losses and element self- and mutual impedances, the tables probably won't give exactly the best feed-line lengths for your array. But if the above assumptions apply, it's very likely that your array will work better using the recommended feed-line lengths. If the assumptions don't describe your array, the table values won't be valid.

Two Four-Element Arrays

The Antenna Book (note 1) describes a feed system for two types of four-element arrays based on a combination of the "current forcing" method and an L network. Information on these arrays, the current forcing method, and practical advice on how to measure the various line sections can be found in Chapter 8 of *The Antenna Book*. The L network can be replaced by two feed lines, resulting in the feed systems shown in Figs 2 and 3. The principle is the same as for the two-element array, although the mathematics are a bit different due to the presence of the λ/4 or 3 λ/4 lines and the difficulty of including the mutual impedances between all elements. The mathematics are described in the next section.

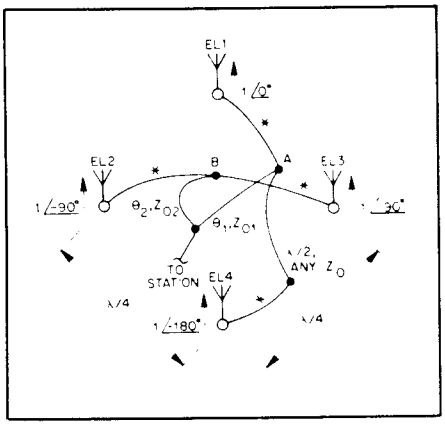

Fig 2—Feed system for the four-square array. Grounds and cable shields have been omitted for clarity. The lines marked "*" are all the same length, have the same Z_0, and are electrically either λ/4 or 3λ/4 long. The other lines are discussed in the text.

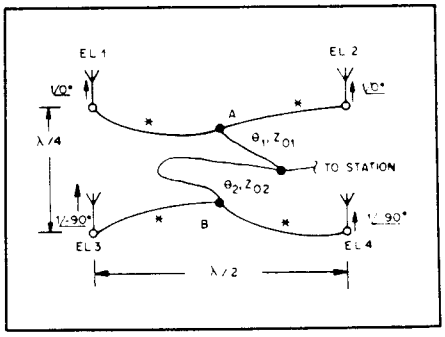

Fig 3—Feed system for a four-element rectangular array. Grounds and cable shields have been omitted for clarity. The lines marked "*" are all electrically 3λ/4 long and have the same Z_0. The other lines are discussed in the text.

Tables 3 and 4 give feed-line lengths for these two arrays. The same restrictions apply to the four-element tables as to the two-element table. They were calculated using modified versions of the BASIC program. These programs, which apply only to the four-square and rectangular arrays, are listed as Programs 2 and 3.

The Mathematics

For an array to work properly, the element currents need to have the correct relationship. So let's first look at the general problem of feeding two loads with a specific ratio of currents (Fig 4). The desired current ratio, I_2/I_1, is a complex number with two parts: magnitude M_{12} and angle ϕ_{12}. Both parts must be correct for the array to work as planned. Assuming for the moment that we know what the load impedance will be, we can write the following equation for feed-line no. 1.

$$V_{in} = I_1 Z_1 \cos \theta_1 + jI_1 Z_{01} \sin \theta_1 \quad \text{(Eq 1)}$$

where

V_{in} = voltage at the input end of the line

I_1 = current at the output end of the line

Z_1 = load impedance at the output end of the line

θ_1 = electrical length of the line in degrees or radians

Z_{01} = characteristic impedance of the line

V_{in}, I_1, and Z_1 are complex

This is the general equation which relates the output current to the input voltage for a lossless transmission line.[3] A similar equation can be written for the second feed line. Since the feed lines are connected together at their input ends, the input voltages are equal, and we can write

$$V_{in} = I_1 Z_1 \cos \theta_1 + jI_1 Z_{01} \sin \theta_1$$

$$V_{in} = I_2 Z_2 \cos \theta_2 + jI_2 Z_{02} \sin \theta_2$$

Rearranging to solve for the current ratio gives

$$\frac{I_2}{I_1} = \frac{(Z_1 \cos \theta_1 + jZ_{01} \sin \theta_1)}{(Z_2 \cos \theta_2 + jZ_{02} \sin \theta_2)} \quad \text{(Eq 2)}$$

This equation can be used to illustrate the problems of feeding unequal load impedances (present in the elements of most arrays). For example, if values that might be found in a 90°-spaced, 90°-fed array are

$Z_1 = 35 - j20\ \Omega$
$Z_2 = 65 + j20\ \Omega$
$Z_{01} = Z_{02} = 50\ \Omega$
$\theta_1 = 90°$, and $\theta_2 = 180°$

then I_2/I_1 would be 0.735 at an angle of $-107°$, not 1 at an angle of $-90°$ as planned. In a real array, because of mutual coupling, the element feed-point impedances are modified by the currents

Table 3

Phasing Line Lengths for a Four Square Array

See Fig 2.

R_s* Ohms	No. Radials per Element	λ/4 Line Z_0, Ohms	Phasing Lines: Z_0, Ohms A	Z_0, Ohms B	Elect Length (Deg) Line A	Elect Length (Deg) Line B
65	4	50	50	50	20.66	166.50
					147.94	204.91
			75	75	13.70	170.60
					158.00	197.70
		75	50	50	No solution	
			75	75	32.03	162.26
					133.53	212.18
54	8	50	50	50	25.82	166.32
					138.25	209.61
			75	75	16.80	170.01
					151.13	202.06
		75	50	50	No solution	
			75	75	45.11	167.22
					115.95	211.72
45	16	50	50	50	34.57	168.71
					123.73	212.98
			75	75	21.15	170.17
					141.45	207.23
		75	50	50	22.36	121.53
					134.77	261.34
			75	75	No solution	
36	∞	50	50	50	No solution	
			75	75	31.37	173.66
					121.79	213.18
		75	50	50	33.55	122.94
					120.01	263.50
			75	75	No solution	

*Self-impedance, including losses.

Table 4

Phasing Line Lengths for a Four Element Rectangular Array

See Fig 3.

R_s* Ohms	No. Radials per Element	3λ/4 Line Z_0, Ohms	Phasing Lines: Z_0, Ohms A	Z_0, Ohms B	Elect Length (Deg) Line A	Elect Length (Deg) Line B
65	4	50	50	50	37.37	155.34
					132.87	179.10
			75	75	24.95	162.22
					150.41	181.47
		75	50	50	No solution	
			75	75	66.82	153.30
					87.34	161.46
54	8	50	50	50	59.02	151.97
					100.15	163.93
			75	75	35.79	157.33
					135.17	175.44
		75	50	50	No solution	
			75	75	No solution	
			75	50	29.44	112.55
					67.01	129.30
45	16	50	50	50	No solution	
			75	75	62.38	152.17
					98.08	159.50
		75	50	50	No solution	
			75	75	No solution	
			75	50	12.61	99.74
					64.34	114.66
36	∞	50	50	50	No solution	
			75	75	No solution	
		75	50	50	52.04	99.08
					176.88	274.80
			75	75	No solution	

*Self-impedance, including losses.

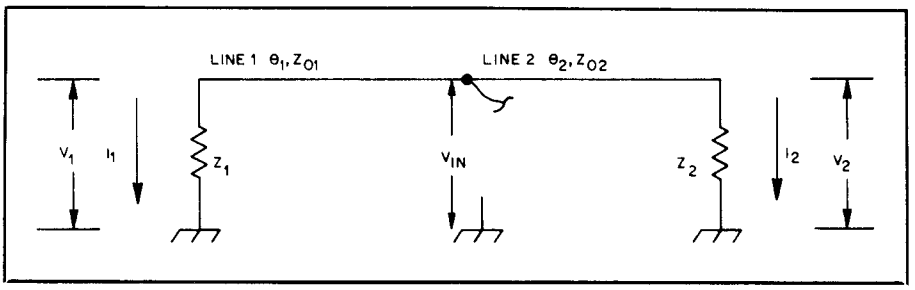

Fig 4—Feeding two load impedances with specific currents. This example assumes that Z1 and Z2 are not affected by mutual coupling.

flowing in the elements. But the element currents are a function of the element feed-point impedances, so Eq 2 can't be used directly to calculate currents in array elements. To write an equation which will do that, we need to modify Fig 4 to account for the effect of mutual coupling (Fig 5). From the diagram,

$$V_1 = I_1 Z_{11} + I_2 Z_{12} \quad \text{(Eq 3)}$$

$$V_2 = I_2 Z_{22} + I_1 Z_{12} \quad \text{(Eq 4)}$$

where

V_n = voltage at the feed point of element n

I_n = current at the feed point of element n

Z_{nn} = self-impedance of element n (the feed-point impedance when the element is totally isolated from all other elements)

Z_{12} = mutual impedance between the elements

All variables are complex

A slightly different form of Eq 1 is

$$V_{in} = V_1 \cos \theta_1 + jI_1 Z_{01} \sin \theta_1 \quad \text{(Eq 5)}$$

and for the second feed line

$$V_{in} = V_2 \cos \theta_2 + jI_2 Z_{02} \sin \theta_2 \quad \text{(Eq 6)}$$

V_1 and V_2 from Eqs 3 and 4 are substituted into Eqs 5 and 6. The right sides of Eqs 5 and 6 are set equal to each other, since V_{in} is the same for both feed lines. Finally, the resulting equation is rearranged to solve for I_2/I_1.

$$\frac{I_2}{I_1} = \frac{Z_{11} \cos\theta_1 - Z_{12} \cos\theta_2 + jZ_{01} \sin\theta_1}{Z_{22} \cos\theta_2 - Z_{12} \cos\theta_1 + jZ_{02} \sin\theta_2} \quad \text{(Eq 7)}$$

This is the same as Eq 2 except that an additional term containing mutual impedance Z_{12} appears in both the numerator and denominator. Given the element self- and mutual impedances and the lengths and impedances of the feed lines, Eq 7 can be used to find the resulting ratio of currents in the elements. The problem we're trying to solve, though, is the other way around: how to find the feed-line lengths, given the current ratio and other factors. Christman described an iterative method of using Eq 7 to solve the problem by beginning with an initial estimate of feed-line lengths, finding the resulting current ratio, correcting the estimate, and repeating until the answer converges on the correct answer.[4] This method gives accurate answers, and I used it for some time. The disadvantage of the approach is that convergence can be slow, and the iterations can actually diverge for some arrays unless the program includes "damping."

Fortunately, an iterative approach isn't necessary, since Eq 7 can be solved directly for feed-line lengths. The method is straightforward, although tedious, and was done using several variable transformations to keep the equations manageable. The details won't be described here. The BASIC programs presented at the end of this paper use the direct solution method, and the validity of the results can be confirmed by substitution into Eq 7.

The feed system can be adapted to certain larger arrays by combining it with the current forcing method described in *The Antenna Book* (see Figs 2 and 3). The basic requirement is to make the *voltages* at points A and B have the proper ratio and phase angle. If this is accomplished, the elements will have correct *currents* because of the properties of the ¼-λ lines. *The Antenna Book* shows the use of an L network to obtain the voltage phase shift; the same thing can be accomplished by using two feed lines of the correct length.

To see how we can use the program to solve the problem, we'll rewrite Eq 2 to apply to the currents and impedances at points A and B:

$$\frac{I_B}{I_A} = \frac{Z_A \cos \theta_1 + jZ_{01} \sin \theta_1}{Z_B \cos \theta_2 + jZ_{02} \sin \theta_2} \quad \text{(Eq 8)}$$

Because $V_A = I_A Z_A$ and $V_B = I_B Z_B$, then

$$\frac{V_B}{V_A} = \frac{Z_B}{Z_A} \frac{I_B}{I_A} = \frac{Z_B}{Z_A} \frac{Z_A \cos \theta_1 + jZ_{01} \sin \theta_1}{Z_B \cos \theta_2 + jZ_{02} \sin \theta_2} \quad \text{(Eq 9)}$$

Note the similarity to Eq 7, which is the equation the program solves for θ_1 and θ_2. We can use the program to solve Eq 9 if we

1) Enter Z_A when it prompts for the self-Z of element 1,

2) Enter Z_B when it prompts for the self-Z of element 2,

3) Enter 0,0 when it prompts for the mutual R, X, and

4) Enter (V_B/V_A) (Z_A/Z_B) when it prompts for the desired current ratio. For the two four-element arrays,

$V_B/V_A = 0 - j1 = 1 \angle -90°$.

The following steps are required to calculate Z_A/Z_B.

1) Measure or estimate the self- and mutual impedances of the elements.

2) Using the self- and mutual impedances and the current ratios, calculate the actual element feed-point impedances. The method is described in *The Antenna Book*.

3) Calculate the impedances looking into the λ/4 or 3λ/4 lines.

4) Where two of the lines are connected,

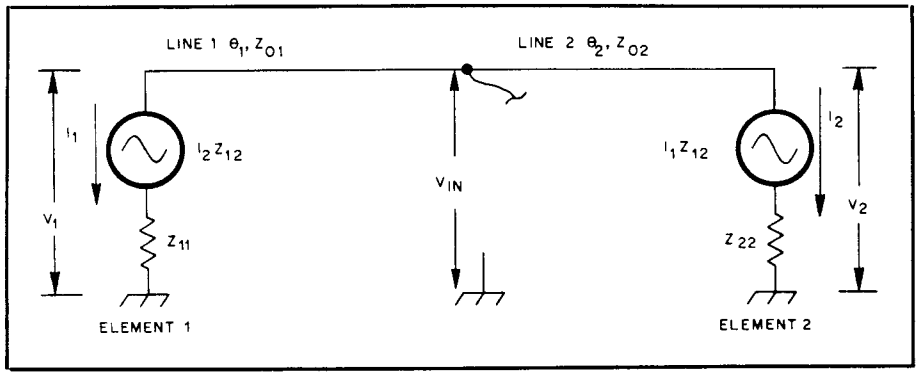

Fig 5—Feeding two antenna elements with specific currents. The voltage sources are added to account for mutual coupling.

calculate the parallel impedance. These will be Z_A and Z_B.

5) Calculate the ratio Z_A/Z_B.

Program 1 has been modified to do these calculations for you. The modified BASIC programs are listed as Programs 2 and 3. [All three programs are available on diskette for the IBM PC and compatibles; see information on an early page of this book.—*Ed.*] You must know the self-impedance of an element (all are assumed to be identical) and the mutual impedances between all elements in order to use the modified programs.

Closing Comments

I first solved Eq 7 for the feed-line lengths several years ago. However, I didn't try to publish the results because of the large amount of explanation which would have to go with it—why the common feed method frequently is disappointing, and explaining the current forcing and L-network feed systems, the role of mutual coupling in phased arrays, etc. I want to thank Jerry Hall, K1TD, for providing the opportunity to explain them in a forum which is readily available to amateurs —Chapter 8 of *The ARRL Antenna Book*.

Your array *will* work better if properly fed. This feed system isn't any more complicated than the one you've probably been using, but it's likely to give you much better results. Try it!

Notes

[1]G. L. Hall, Ed, *The ARRL Antenna Book*, 15th ed. (Newington: ARRL, 1988), Chap 8.

[2]J. Sevick, "The Ground-Image Vertical Antenna," *QST*, Jul 1971, pp 16-19, 22. Also "The W2FMI Ground-Mounted Short Vertical," *QST*," Mar 1973, pp 13-18, 41. (Summary information from these articles is presented graphically in Fig 23, p 8-23 of *The Antenna Book*—see note 1).

[3]*Reference Data for Radio Engineers*, 5th ed. (Howard W. Sams & Co, 1968).

[4]A. Christman, "Feeding Phased Arrays: An Alternative Method," *Ham Radio*, May 1985, p 58 and Jul 1985, p 74.

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User Notes – March 23, 2005

A number of arrays have been built and tested using the method described in the article. All worked as predicted. These arrays can be and have been modeled, and the models also show performance as predicted.

The original article included a BASIC code listing for all the programs. You can get the complete original GWBASIC source code and executable programs by downloading SIMPFEED.ZIP from ftp://eznec.com/pub/. A new Windows program with advanced features is available which uses the principles detailed in the article. Following is a description of the new program:

Simpfeed

Simpfeed is a Windows application for designing the feed networks for several array types. It requires **EZNEC** or the free **EZNEC** demo program (http://eznec.com/demoinfo.htm) to provide the feedpoint impedance data it needs. **EZNEC** can also be used to model the resulting all-transmission line fed array. Simpfeed includes a manual with a detailed example of its use. Simpfeed is available at http://eznec.com/Amateur/Articles/SimpfeedZip.exe. This is a self-extracting ZIP file which contains the program and manual. No special installation is required. However, it does require the Visual Basic 6 runtime files. These are present on all Windows ME, 2000, and XP systems, and they're installed with **EZNEC** or the **EZNEC** demo program. So on some systems it might be necessary to install **EZNEC** or the **EZNEC** demo program before running Simpfeed.

After downloading and running the Simpfeed Zip file, put the resulting files into the same directory. Start Simpfeed by running Simpfeed.exe. Click Help to open the manual. The example in the manual takes you through the design of a four square feed system and its modeling with **EZNEC** or the **EZNEC** demo program.

Simpfeed can be freely copied and distributed, provided that no fee is charged for it. See the disclaimers in the Introduction topic of the Simpfeed manual.